Spin chirality induced by the Dzyaloshinskii-Moriya interaction and the polarized neutron scattering

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We discuss the influence of the Dzyaloshinskii-Moriya (DM) interaction in the Heizenberg spin chain model for the observables in the polarized neutron scattering experiments. We show that different choices of the parameters of DM interaction may leave the spectrum of the problem unchanged, while the observable spin-spin correlation functions may differ qualitatively. Particularly, for the uniform DM interaction one has the incommensurate fluctuations and polarization-dependent neutron scattering in the paramagnetic phase. We sketch the possible generalization of our treatment to higher dimensions.

Since the works by Dzyaloshinskii and Moriya[1], the antisymmetric spin exchange interaction plays an important role in the physics of condensed matter. Being introduced for the explanation of the weak ferromagnetism in antiferromagnets without center of inversion, the Dzyaloshinskii-Moriya (DM) interaction is found nowadays in various problems of magnetism and statistical physics.

Being the relativistic effect, the magnitude of DM interaction, D, is generally expected to be small in comparison with the usual symmetric superexchange, J.[1,2] In some compounds, however, this interaction can attain a sizeable value. For instance, one has $D/J = 0.18$ in the hexagonal perovskite $CsCuCl_3$[3] and $D/J \approx 0.05$ in copper benzoate.[4]

Remarkably, the DM interaction in the latter compounds takes place in the quasi-one-dimensional spin subsystems. On this reason, we are primarily concerned below with the one-dimensional (1D) situation of a quantum spin chain.

Generally, the DM interaction between two spins, $\mathbf{S}_{1,2}$, is written as $\mathbf{D}(\mathbf{S}_1 \times \mathbf{S}_2)$ with an axial DM vector $\mathbf{D}$. In a chain, $\mathbf{D}$ may spatially vary both in direction and magnitude, however, the symmetry arguments usually rule out most of the possibilities and confine the theoretical discussion to two principal cases. The first one is the uniform DM interaction, $\mathbf{D} = const$ over the system.[3] The second case is the staggered DM interaction,[4] with antiparallel $\mathbf{D}$ on adjacent bonds.

Among the other studies, we should mention the discussion of the XY spin chains with randomly distributed values of D[5] and the growth models with imaginary uniform $D = i\lambda$ leading to non-Hermitian Hamiltonian.[6] A model of XY spin chain with a ternary DM interaction was introduced and solved recently.[7]

In a present paper, we deal mostly with two above cases, uniform and staggered DM interaction. We consider also a model of a non-ideal lattice, where one finds, say, an almost uniform situation with one possible DM value, $\mathbf{D}$, taking place on a chain fragment of average length l_1, and another value, $-\mathbf{D}$, on a chain fragment of length l_2, while $l_{1,2} \gg 1$. The situation is then described in probabilistic terms. We show that, being the situation uniform, staggered or random, the spectrum of the Heisenberg chain with the DM interaction is equivalent to one of XXZ spin model and is computed exactly for spin 1/2. Of course the observable susceptibilities can differ crucially, as we demonstrate below.

Therefore we extend the previous result by Alcaraz and Wreszinski, that the Heisenberg 1D model with the uniform DM interaction is reduced to XXZ spin exchange model and is exactly solvable.[8]

For the uniform and almost uniform DM interaction, we show that the observable spin-spin correlation function possess an incommensurate structure. This incommensurability phenomenon was noted previously for the XY spin chain model and uniform antisymmetric spin interaction.[9] We show further that in this case the spin susceptibility tensor $\chi^{\alpha\beta}$ acquires an antisymmetric part. This leads to the appearance of the polarization-dependent part of the neutron scattering cross-section, which makes possible the direct observation of the direction and the value of the DM vector $\mathbf{D}$. Our results are applicable in the absence of the long-range magnetic order in the system. They also can be generalized towards higher-dimensional situation, as discussed below. Therefore our treatment may provide an explanation of the earlier experiments in cubic ferromagnet MnSi,[10] where the DM-induced incommensurability of the magnetic fluctuations and polarization dependence of the neutron scattering were observed both below and above the Curie ordering temperature.

We consider the spin chain Hamiltonian of the form

$$\mathcal{H} = \sum_{l=1}^{L} (J\mathbf{S}_l\mathbf{S}_{l+1} + \mathbf{D}_l[\mathbf{S}_l \times \mathbf{S}_{l+1}]) \qquad (1)$$

with AF Heisenberg coupling J and the Dzyaloshinskii-Moriya term $\mathbf{D}_l$. We choose the vector $\mathbf{D}_l$ to be directed along the $z-$axis.

We observe that $\mathcal{H}$ is simplified upon a canonical transformation $\mathcal{H} \to e^{-iU}\mathcal{H}e^{iU}$ with

$$U = \sum_{l=1}^{L} \alpha_l S_l^z, \qquad \alpha_l = \sum_{i=1}^{l-1} \tan^{-1}(D_i/J), \qquad (2)$$

and $\alpha_1 = 0$. Note that this transformation works for all values of S. The periodic boundary conditions (BC) require $\alpha_{L+1} = 0 \bmod 2\pi$ which relation is not generally satisfied. However, in the thermodynamic limit $L \to \infty$ the influence of BC can be negliged.[8] Introducing $S_j^{\pm} = S_j^x \pm iS_j^y$, one can easily see that

$$\widetilde{S}_l^{\pm} \equiv e^{-iU} S_l^{\pm} e^{iU} = S_l^{\pm} e^{\mp i\alpha_l}, \quad \widetilde{S}_l^z = S_l^z, \tag{3}$$

and our choice of the coefficients α_l removes the antisymmetric part of the Hamiltonian, $[\widetilde{\mathbf{S}}_l \times \widetilde{\mathbf{S}}_{l+1}]$. We consider below two principal possibilities: the "uniform" situation $D_l = J\tan\delta$ and the "staggered" one $D_l = (-1)^l J \tan\delta$. In both cases the Hamiltonian is reduced to the XXZ model :

$$\mathcal{H} = \sum_{l=1}^{L} (J^x(\widetilde{S}_l^x \widetilde{S}_{l+1}^x + \widetilde{S}_l^y \widetilde{S}_{l+1}^y) + J \widetilde{S}_l^z \widetilde{S}_{l+1}^z) \tag{4}$$

with $J^x = \sqrt{J^2 + D_l^2}$ independent of l.

Our subsequent discussion is based on the observation, that the DM interaction results in two effects for the observable susceptibility of the system. First effect is the modification of the spectrum, as seen in the equivalent Hamiltonian (4). The appearance of the "easy-plane" anisotropy ($J^x > J$), however, does not lead to a gap in the spectrum. The exact solution of (4) for $S = 1/2$ shows[11] that the correlation functions $\langle \widetilde{S}_l^x \widetilde{S}_m^x \rangle \sim |l-m|^{-\nu}$ and $\langle \widetilde{S}_l^z \widetilde{S}_m^z \rangle \sim |l-m|^{-1/\nu}$ with $\nu = 1 - |\delta|/\pi$. Since the value of DM exchange is expected to be small, $|D_l| \ll J$, the long-distance decay of the above correlation functions is described to a good accuracy by the "isotropic" Heisenberg situation, with $\nu = 1$. The second effect of DM interaction for the observables is the explicit dependence of the relation (3) between the new and old spin variables on the values of D_l. We focus our attention below on the latter effect, which leads to the qualitative changes in the experimentally observable susceptibilities.

The two-time Green's function for the operators A and B is defined as $\chi_{AB}(t) = -i\theta(t)\langle [A(t), B] \rangle$ where $[\ldots,\ldots]$ stands for a commutator and $\theta(t) = 1$ at $t > 0$.

Upon the "twist" e^{iU} the z-component of spin operators remains unchanged, and one has for the longitudinal zz susceptibility $\chi_{lm}^{zz}(t) = -i\theta(t)\langle [S_l^z(t), S_m^z] \rangle = -i\theta(t)\langle [\widetilde{S}_l^z(t), \widetilde{S}_m^z] \rangle \equiv \mathcal{G}_{lm}^{\parallel}(t)$. Therefore the observable $\chi_{lm}^{zz}(t)$ has a commensurate antiferromagnetic modulation.

The expressions for the transverse spin susceptibility are more complicated. It is convenient to introduce the matrix[12]

$$\chi_{lm}^{\perp}(t) = -i\theta(t) \left[\begin{array}{ll} \langle [S_l^x(t), S_m^x] \rangle, & \langle [S_l^x(t), S_m^y] \rangle \\ \langle [S_l^y(t), S_m^x] \rangle, & \langle [S_l^y(t), S_m^y] \rangle \end{array} \right], \tag{5}$$

in the initial system (1). In the simpler "twisted" system (4) we have $-i\theta(t)\langle [\widetilde{S}_l^x(t), \widetilde{S}_m^x] \rangle = -i\theta(t)\langle [\widetilde{S}_l^y(t), \widetilde{S}_m^y] \rangle \equiv \mathcal{G}_{lm}^{\perp}(t)$ and $\langle [\widetilde{S}_l^x(t), \widetilde{S}_m^y] \rangle = \langle [\widetilde{S}_l^y(t), \widetilde{S}_m^x] \rangle = 0$. Returning back to quantities $\langle [S_l^\alpha(t), S_m^\beta] \rangle$ with the use of (3), we get

$$\chi_{lm}^{\perp}(t) = \mathcal{G}_{lm}^{\perp}(t) \left[\begin{array}{ll} \cos\alpha_{l,m}, & -\sin\alpha_{l,m} \\ \sin\alpha_{l,m}, & \cos\alpha_{l,m} \end{array} \right] \tag{6}$$

with $\alpha_{l,m} = \alpha_l - \alpha_m$.

For later comparison, it is worth to consider first the case of the staggered DM interaction.[4] We have $D_l = (-1)^l J\tan\delta$ and $\alpha_{l,m} = ((-1)^l - (-1)^m)\delta/2$. In this case we write $\cos\alpha_{l,m} = \cos^2(\delta/2) + (-1)^{l-m}\sin^2(\delta/2)$ and $\sin\alpha_{l,m} = ((-1)^l - (-1)^m)(\sin\delta)/2$. Clearly, the off-diagonal components $\chi_{lm}^{xy}(t)$ of the matrix (6) do not depend on the difference $(l-m)$ only and the two-momenta Fourier transform $A(q,q',\omega) = \int dt \sum_{lm} e^{iql - iq'm - i\omega t} A_{lm}(t)$ should be introduced. Then we obtain the off-diagonal components in the form $\chi^{xy}(q,q',\omega) = (\mathcal{G}^{\perp}(q,\omega) - \mathcal{G}^{\perp}(q-\pi,\omega)) \sum_\tau \delta(q - q' - \pi + \tau)$ where $\sum_\tau$ stands for the sum over all vectors $\tau = 2\pi n$ of the reciprocal lattice. However, apparently in all physical observables one finds the symmetrized form of the susceptibility ($q = q'$) and the off-diagonal terms in the matrix $\chi^{\perp}$ vanish. Therefore in the case of staggered DM interaction one is left with the diagonal component of the matrix $\chi^{\perp}(q,\omega)$ of the form :

$$\begin{aligned} \chi^{\perp}(q,\omega) &= \mathcal{G}^{\perp}(q,\omega)\cos^2(\delta/2) \\ &+ \frac{1}{2}[\mathcal{G}^{\perp}(q-\pi,\omega) + \mathcal{G}^{\perp}(q+\pi,\omega)]\sin^2(\delta/2) \end{aligned} \tag{7}$$

We see that the regions of the AF and the ferromagnetic fluctuations are mixed in the observable susceptibility. A consequence of this feature[4] is the anomalous temperature behavior of the uniform static susceptibility $\chi^{xx}(0,0) = \chi^{yy}(0,0)$ for the AF chain (see also the discussion after Eq.(5.3) in the original Moriya's paper[1]). It is known[13] that in the Heisenberg $S = 1/2$ chain one has $\mathcal{G}^{\perp}(0,0) \sim J^{-1}$ and $\mathcal{G}^{\perp}(\pi,0) \sim T^{-2+\nu}$, therefore $\chi^{zz}(0,0) \sim J^{-1}$ and

$$\chi^{xx}(0,0) = \chi^{yy}(0,0) \sim J^{-1}[const + \delta^2 (J/T)^{1+|\delta|/\pi}] \tag{8}$$

The Eq. (8) has a simple physical meaning. Indeed, in the considered case the operator U "cants" the local coordinate frames by an angle $\pm\delta/2$. It leads effectively to the non-compensated spin $\Delta S = \delta/4$ in the $x-y$ plane. Being the spins ΔS free, it would then lead to the Curie law for the susceptibility $\chi \sim \langle \Delta S^2 \rangle / T$. The 1D character of the interacting spin system results in the nontrivial exponent in the $T-$dependence of this term (cf. also[4]).

At the same time, the above transfer of the spectral weight, Eq. 7, is apparently negligible to be observed in the neutron scattering experiments.

On the other hand, for the "uniform" DM interaction $D_l = D$ we have $\alpha_{l,m} = (l-m)\delta$. It results in the incommensurability of the transverse spin correlations. Fourier transforming Eq. (6), we obtain

$$\chi^{\perp}(q,\omega)=\frac{1}{2}\mathcal{G}^{\perp}(q+\delta,\omega)\left[\begin{array}{cc}1, & i\\ -i, & 1\end{array}\right] +\frac{1}{2}\mathcal{G}^{\perp}(q-\delta,\omega)\left[\begin{array}{cc}1, & -i\\ i, & 1\end{array}\right] \quad (9)$$

Let us discuss the physical consequences of this expression. Evidently, $\chi^{xx}(0,0)\sim const$ in this case and the presence of the "uniform" DM interaction is not revealed by the measurements of the temperature dependence of the uniform static susceptibility. Much more interesting are the implications of (9) for the polarized neutron scattering experiments. The basic quantity here is neutron scattering cross-section, which is connected to the Green's function $\chi^{\alpha\beta}(q,\omega)$ of the spin system. It is convenient to write $\chi^{\alpha\beta}=\chi^{\alpha\beta}_S-i\chi^{\alpha\beta}_A$, with the symmetric and antisymmetric tensors, $\chi^{\alpha\beta}_S$ and $\chi^{\alpha\beta}_A$, respectively. Up to fundamental constants, we have[14] :

$$\frac{d^2\sigma(q,\omega)}{d\Omega d\omega}\sim N(-\omega)[Im\chi^{\alpha\beta}_S(q,\omega)(\delta^{\alpha\beta}-\widehat{Q}^{\alpha}\widehat{Q}^{\beta}) +Im\chi^{\alpha\beta}_A(q,\omega)\epsilon_{\alpha\beta\gamma}\widehat{Q}^{\gamma}(\widehat{\mathbf{Q}}\mathbf{P}_0)], \quad (10)$$

where $N(\omega)$ is the Planck function, the unit vector $\widehat{\mathbf{Q}}=\mathbf{Q}/Q$ is directed along the neutron's momentum transfer $\mathbf{Q}$, $\epsilon_{\alpha\beta\gamma}$ is totally antisymmetric tensor, $\mathbf{P}_0$ is the incident neutron's polarization and q is the on-chain projection of $\mathbf{Q}$.

From (10) we see that if the whole crystal is characterized by Dzyaloshinskii vector $\mathbf{D}$ (uniform situation, Eq. 9), then the polarization-dependent part of cross-section is non-zero and is given by

$$\frac{d^2\sigma_1}{d\Omega d\omega}\sim(\mathbf{D}\widehat{\mathbf{Q}})(\widehat{\mathbf{Q}}\mathbf{P}_0)Im\frac{\mathcal{G}^{\perp}(q+\delta,\omega)-\mathcal{G}^{\perp}(q-\delta,\omega)}{2D}. \quad (11)$$

A certain subtlety should be discussed here. Under the parity transformation we have $q\to-q$, $\mathbf{D}\to\mathbf{D}$, $\mathbf{P}_0\to\mathbf{P}_0$. At the first glance $\delta\to\delta$ and thus $d^2\sigma_1/d\Omega d\omega$ changes the sign, as it should not be. An inspection of (2), (6) (9) shows however that the quantity δ in (11) appears as the differential of α_l. The latter object is the sum of the phases δ_j over the bonds j to the left of l. Hence we have under the parity transformation $\alpha_l\to-\alpha_l+const$ and $\delta\to-\delta$ in (11), which restores the desired property of the cross-section.

The contribution of the symmetric part of $\chi^{\alpha\beta}$ to $d^2\sigma(q,\omega)/d\Omega d\omega$ in the considered case of uniform $\mathbf{D}$ is two-fold. One still has the commensurate fluctuations of spin components $S^z\|\mathbf{D}$, with a peak at the AF position. At the same time, the DM interaction splits the AF peak related to transverse fluctuations into two peaks of the weight 1/2, Eq. 9. The relative weights of these two structures depend on the direction of $\mathbf{Q}$ as follows

$$\frac{d^2\sigma_2(q,\omega)}{d\Omega d\omega}\sim(1+\widehat{Q}^2_z)\,Im\frac{\mathcal{G}^{\perp}(q+\delta,\omega)+\mathcal{G}^{\perp}(q-\delta,\omega)}{2} +(1-\widehat{Q}^2_z)\,Im\,\mathcal{G}^{\|}(q,\omega). \quad (12)$$

Note that at low temperatures, the incommensurate peaks have more singular behavior according to our discussion after Eq. (4).

An important thing to be stressed here is the following. It is known that the incommensurate long-range magnetic structures may arise due to the competing interactions in the spin system. In this case one expects that all three diagonal components of the spin susceptibility $\chi^{\alpha\alpha}$ are peaked *in the paramagnetic region* at the same incommensurate wave-vector. The off-diagonal components of $\chi^{\alpha\beta}$ are absent. This is fairly different from the picture described above, Eqs. 11, 12. Hence the experimental observation of the incommensurability phenomenon in the paramagnetic phase, accompanied by the polarization dependence of the neutron scattering cross-section could serve as an indication to the presence of the uniform DM interaction. Remarkably, the value and the direction of the pseudo-vector $\mathbf{D}$ can be, in principle, determined this way.

In reality, however, the macroscopic sample is rarely uniform and it should be expected to split to domains with different directions of $\mathbf{D}$. To account for this situation, it is instructive to analyze a model where the value of the DM interaction D_l takes randomly two values $\pm J\tan\delta$.

Consider first the oversimplified case when $\langle\!\langle D_l\rangle\!\rangle=0$ and $\langle\!\langle D_lD_m\rangle\!\rangle=0$ for $l\neq m$, here $\langle\!\langle\ldots\rangle\!\rangle$ denotes averaging over the realizations. In this case the spectrum is still defined by Eq.(4), and χ^{zz} is given by the above expression. At the same time, one can easily show that the averaged susceptibility $\chi^{\perp}$ has a diagonal form and exhibits an exponential decay of correlations :

$$\langle\!\langle\chi^{\perp}_{lm}(t)\rangle\!\rangle=\mathcal{G}^{\perp}_{lm}(t)\exp(-|l-m|/l_*) \quad (13)$$

with the correlation length $l_*=-1/\ln(\cos\delta)\sim\delta^{-2}$.

Now consider a more realistic situation when one still has $\delta_j=\pm\delta$, but the signs of δ_j on the adjacent bonds are correlated. The diagonal and off-diagonal parts of the matrix (6) are given, respectively, by the real and imaginary part of the average

$$\langle\!\langle\exp i\alpha_{l,m}\rangle\!\rangle\equiv\sum_{\{\delta_j\}}p(\delta_1,\ldots,\delta_L)\exp(i\sum_{k=m}^{l-1}\delta_k). \quad (14)$$

We assume that the joint distribution function $p(\delta_1,\ldots,\delta_n)$ has a Markovian character, $p(\delta_1,\ldots,\delta_n)=p(\delta_1,\ldots,\delta_{n-1})\hat{p}(\delta_n|\delta_{n-1})$. In this physically important case we arrive at the dichotomous Markovian noise δ_j with a discrete "time" j.[15] We set $\langle\!\langle\delta_j\rangle\!\rangle=\delta d$ which defines the on-site ("equilibrium") probability as $p_0=(\frac{1+d}{2},\frac{1-d}{2})$. The matrix $\hat{p}(\delta_n|\delta_{n-1})$ satisfies the "conservation laws" for the total and equilibrium probabilities, $(1,1)\cdot\hat{p}(\delta_n|\delta_{n-1})=(1,1)$ and $\hat{p}(\delta_n|\delta_{n-1})\cdot p_0=p_0$, respectively. These equalities fix $\hat{p}(\delta_n|\delta_{n-1})$ in the form

$$\hat{p}=\left[\begin{array}{cc}1-x(1-d), & x(1+d)\\ x(1-d), & 1-x(1+d)\end{array}\right]$$

for all n, which corresponds to the following correlator on the adjacent sites : $\langle\langle \delta_j \delta_{j+1} \rangle\rangle - \langle\langle \delta_j \rangle\rangle^2 = \delta^2(1-d^2)(1-2x)$. The latter equalities mean that the absence of correlations corresponds to $x = 1/2$ and the correlation lengths for positive and negative sequences of δ_j are $1/l_{1,2} = x(1 \mp d)$. Introducing the matrix $\mathcal{D} = diag(e^{i\delta}, e^{-i\delta})$, the quantity (14) is represented as a product $(1,1) \cdot (\mathcal{D} \cdot \hat{p})^{l-m-1} \cdot \mathcal{D} \cdot p_0$, which is evaluated using the multiplication rules for the Pauli matrices. After straightforward, though tedious, calculation, we obtain the average (14) in general form, which is somewhat simplified in two principal cases of small δ : i) $\delta \ll x \sim 1$ and ii) $\delta \sim x \ll 1$. In the first case, keeping the terms of order of δ^2, we have

$$\begin{aligned} \langle\langle e^{i\alpha_{m+n,m}} \rangle\rangle \simeq & \exp[n(i\delta d - \delta^2 a_1) + \delta^2 a_2] \\ & - \delta^2 a_2 (1-2x)^n \exp[(n-1)(-i\delta d + \delta^2 a_1)] \end{aligned} \quad (15)$$

here $a_1 = (1-x)(1-d^2)/(2x)$ and $a_2 = (1-d^2)(1-2x)/(4x^2)$. We see that the incommensurability wave vector is defined by the average on-bond value δd. Note that Eqs. (7), (13) are recovered at $d=0$, $x=1$ and $d=0$, $x = 1/2$, respectively.

When $x \sim \delta \ll 1$, we come to a more complicated situation. We have in this case

$$\langle\langle e^{i\alpha_{m+n,m}} \rangle\rangle \simeq e^{-xn} \left[\cosh bn + \frac{\sinh bn}{b}(x + i\delta d) \right] \quad (16)$$

with $b = \sqrt{x^2 + 2ix\delta d - \delta^2}$. We return to Eq. (9) at $x=0$ and $d=1$.

It should be stressed that at $d = 0$ one has $Im \langle\langle e^{i\alpha_{m+n,m}} \rangle\rangle = 0$ both in Eqs. (15), (16) and in general case. It corresponds to the fact that the off-diagonal components of susceptibility $\chi^{\perp}$, Eq. (6), vanish in the system with zero average Dzyaloshinskii vector $\langle\langle \mathbf{D} \rangle\rangle$. As a result, the observable cross-section is polarization-independent. The position of maximum of the transverse spin fluctuations, though, may be incommensurate one for the almost uniform DM interaction, as seen from (16) at $d = 0$ and $x \to 0$.

Now we discuss the possible generalization of our approach to a higher dimensional case. Consider a planar system with spins $\mathbf{S}_{lm}$ labeled by two indices. The interaction between spins takes place in two directions, and we write the corresponding quantities as $J_{lm}^{(\alpha)}$ and $\mathbf{D}_{lm}^{(\alpha)}$, with $\alpha = x, y$. For simplicity we consider the case when the vectors $\mathbf{D}_{lm}^{(\alpha)}$ lie along one direction, with possible variation in their magnitude. We introduce then two angles, $\delta_{lm}^x = \tan^{-1}(D_{lm}^{(x)}/J_{lm}^{(x)})$ and $\delta_{lm}^y = \tan^{-1}(D_{lm}^{(y)}/J_{lm}^{(y)})$. We are interested to arrive to a symmetrized Hamiltonian, similar to (4), by making the transformation with $U = \sum_{l,m=1}^{L} \alpha_{lm} S_{lm}^z$. One can show that this transformation is possible if and only if $\delta_{l,m+1}^x - \delta_{lm}^x = \delta_{l+1,m}^y - \delta_{lm}^y$. In this case α_{lm} is uniquely defined and may be written in a form $\alpha_{lm} = \sum_{j=1}^{l-1} \delta_{j,1}^x + \sum_{j=1}^{m-1} \delta_{l-1,j}^y$. The latter relations are not surprizing when we note that in the continuum limit they read as $\nabla \times \vec{\delta}_{\mathbf{r}} = 0$, while $\vec{\delta}_{\mathbf{r}} = (\delta_{\mathbf{r}}^x, \delta_{\mathbf{r}}^y) = \nabla \alpha_{\mathbf{r}}$. In particular, the two-dimensional situation with $D_{lm}^{(x)} = D$ and $D_{lm}^{(y)} = 0$ allows this transformation. The calculation of observables and the generalization to a three-dimensional case are done in the way similar to the above one.

In conclusion, we discuss the observables in the spin system with the antisymmetric DM interaction. We show that in one spatial dimension the exactly found spectrum of such problem may coincide for different choices of the parameters of DM interaction. Despite this fact, the observable spin-spin correlation functions may differ crucially, and we discuss this feature with the application to the neutron scattering experiments. In particular, the incommensurability and the polarization dependence of the neutron scattering may be used for the determination of the value of the uniform DM interaction.

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